

Bits, Bytes, and Integers

CSE4009: System Programming

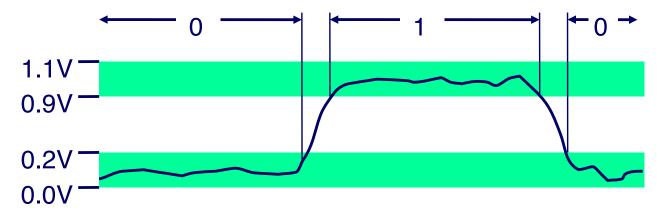
Woong Sul

Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
 - Representation: unsigned and signed
 - Conversion, casting
 - Expanding, truncating
 - Addition, negation, multiplication, shifting
 - Summary
- Representations in memory, pointers, strings

Everything is Bits

- Each bit is 0 or 1
- By encoding/interpreting sets of bits in various ways
 - Computers determine what to do (instructions)
 - ... and represent and manipulate numbers, sets, strings, etc...
- Why bits? Electronic Implementation
 - Easy to store with bistable elements
 - Reliably transmitted on noisy and inaccurate wires



Example: Counting in Binary

- Base 2 Number Representation
 - Represent 15213₁₀ as 11101101101101₂
 - Represent 1.20₁₀ as 1.001100110011[0011]...₂
 - Represent 1.5213 X 10⁴ as 1.1101101101101₂ X 2¹³



Encoding Byte Values

- Byte = 8 bits
 - Binary 00000002 to 111111112
 - Decimal: 0₁₀ to 255₁₀
 - Hexadecimal 00₁₆ to FF₁₆
 - Base 16 number representation
 - Use characters '0' to '9' and 'A' to 'F'
 - Write FA1D37B₁₆ in C as
 - 0xFA1D37B
 - 0xfa1d37b

Hex Decimal Binary

0	0	0000
1	1	0001
2 3	2	0010
	I 3	0011
4	4	0100
5	I 5	0101
6	6	0110
7	7	0111
R	8	1000
9	9	1001
A	10	1010
В	11	1011
С	12	1100
D	13	1101
E	14	1110
F	15	1111

Data Representations Example

C Data Type	Typical 32-bit	Typical 64-bit	x86-64
char	1	1	1
short	2	2	2
int	4	4	4
long	4	8	8
float	4	4	4
double	8	8	8
long double	-	-	10/16
pointer	4	8	8

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Boolean Algebra

- Developed by George Boole in 19th Century
 - Algebraic representation of logic
 - Encode "True" as 1 and "False" as 0

And

Or

■ A&B = 1 when both A=1 and B=1

 \blacksquare A | B = 1 when either A=1 or B=1

Not

Exclusive-Or (Xor)

■ ~A = 1 when A=0

■ A^B = 1 when either A=1 or B=1, but not both

General Boolean Algebras

- Operate on Bit Vectors
 - Operations applied bitwise

```
01101001 01101001 01101001

& 01010101 | 01010101 ^ 01010101 ~ 01010101

01000001 01111101 00111100 1010101
```

All of the Properties of Boolean Algebra Apply

Example: Representing & Manipulating Sets

Representation

• Width w bit vector represents subsets of {0, ..., w−1}

```
    a<sub>j</sub> = 1 if j ∈ A
    01101001 { 0, 3, 5, 6 }
    76543210
    01010101 { 0, 2, 4, 6 }
    76543210
```

Manipulation

&	Intersection	01000001	{ 0, 6 }
	Union	01111101	{ 0, 2, 3, 4, 5, 6 }
٨	Symmetric difference	00111100	{ 2, 3, 4, 5 }
~	Complement	10101010	{ 1, 3, 5, 7 }

Bit-Level Operations in C

- Operations & | ~ ^ Available in C
 - Apply to any "integral" data type
 long, int, short, char, unsigned
 - View arguments as bit vectors
 - Arguments applied bit-wise
- Examples (char data type)
 - $\sim 0x41 \rightarrow 0xBE$ $\sim 010000012 \rightarrow 101111102$
 - $\sim 0 \times 00 \rightarrow 0 \times FF$ $\sim 0000000002 \rightarrow 1111111112$
 - $0x69 \& 0x55 \rightarrow 0x41$ $011010012 \& 010101012 \rightarrow 010000012$
 - $0x69 \mid 0x55 \rightarrow 0x7D$ $011010012 \mid 010101012 \rightarrow 011111012$

Contrast: Logical Operations in C

- Contrast to Logical Operators
 - &&, ||, !
 - View 0 as "False"
 - Anything non-zero as "True"
 - Always return 0 or 1
 - Early termination
- Examples (char data type)

```
!0x41 \rightarrow 0x00

!0x00 \rightarrow 0x01

!!0x41 \rightarrow 0x01

0x69 && 0x55 \rightarrow 0x01

0x69 | | 0x55 \rightarrow 0x01

p && *p (avoids null pointer access)
```

Watch out for && vs. & (and || vs. |)... one of the more common oopsies in C programming

Shift Operations

- Left Shift: x << y
 - Shift bit-vector **x** left **y** positions
 - Throw away extra bits on left
 - Fill with 0's on right
- Right Shift: x >> y
 - Shift bit-vector x right y positions
 - Throw away extra bits on right
 - Logical shift
 - Fill with 0's on left
 - Arithmetic shift
 - Replicate most significant bit on left
- Undefined Behavior
 - Shift amount < 0 or ≥ word size

Argument x	01100010	
<< 3	00010000	
Log. >> 2	00011000	
Arith. >> 2	00011000	

Argument x	10100010
<< 3	00010000
Log. >> 2	00101000
Arith. >> 2	11 101000

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Encoding Integers

Unsigned

$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$

short int
$$x = 15213$$
;
short int $y = -15213$;

Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

Sign Bit

C short 2 bytes long

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
У	-15213	C4 93	11000100 10010011

- Sign Bit
 - For 2's complement, most significant bit indicates sign
 - 0 for nonnegative
 - 1 for negative

Numeric Ranges

Unsigned Values

- *UMin* = 0 000...0
- $\bullet UMax = 2^w 1$ 111...1

• Two's Complement Values

- $TMin = -2^{w-1}$ 100...0
- $TMax = 2^{w-1} 1$ 011...1
- Other Values
 - Minus 1 111...1

Values for W = 16

	Decimal	Hex	Binary
UMax	65535	FF FF	11111111 11111111
TMax	32767	7F FF	01111111 11111111
TMin	-32768	80 00	10000000 00000000
-1	-1	FF FF	11111111 11111111
0	0	00 00	00000000 00000000

Values for Different Word Sizes

	W			
	8	16	32	64
UMin	0	0	0	0
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

Observations

- Umin = 0
- |TMin| = TMax + 1
 - Asymmetric range
- UMax = 2 * TMax + 1

- C Programming
 - #include <limits.h>
 - Declares contants, e.g.,
 - ULONG_MAX
 - LONG_MAX
 - LONG_MIN
 - Values are platform-specific

Unsigned & Signed Numeric Values

Χ	B2U(<i>X</i>)	B2T(<i>X</i>)
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	-8
1001	9	- 7
1010	10	-6
1011	11	- 5
1100	12	-4
1101	13	-3
1110	14	-2
1111	15	-1

Equivalence

Same encodings for nonnegative values

Uniqueness

- Every bit pattern represents unique integer value
- Each representable integer has unique bit encoding

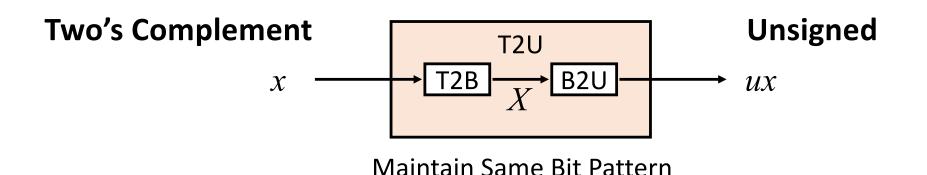
⇒ Can Invert Mappings

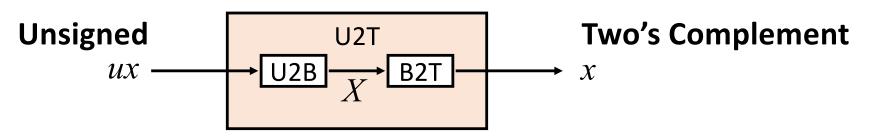
- U2B(x) = B2U⁻¹(x)
 - Bit pattern for unsigned integer
- $T2B(x) = B2T^{-1}(x)$
 - Bit pattern for two's comp integer

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Mapping Between Signed & Unsigned





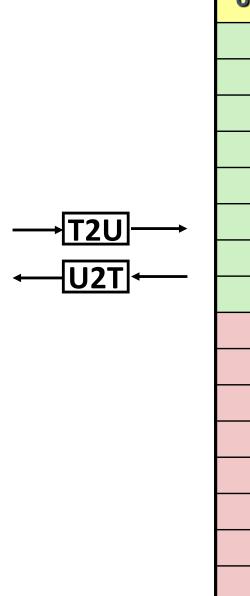
Maintain Same Bit Pattern

Mappings between unsigned and two's complement numbers:
 Keep bit representations and reinterpret

Mapping Signed ↔ Unsigned

Bits
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111

Signed
0
1
2
3
4
5
6
7
-8
-7
-6
-5
-4
-3
-2
-1

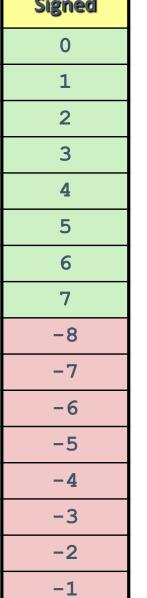


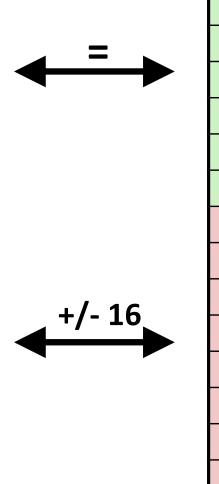
Unsign	ed
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	

Mapping Signed ↔ **Unsigned**

Bits
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111

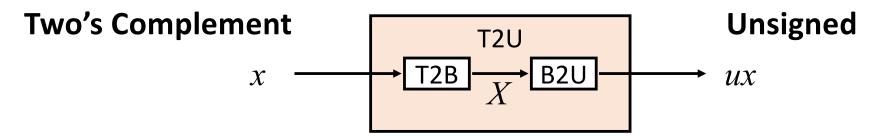
Signed
0
1
2
3
4
5
6
7
-8
-7
-6
- 5
-4
-3
-2
-1



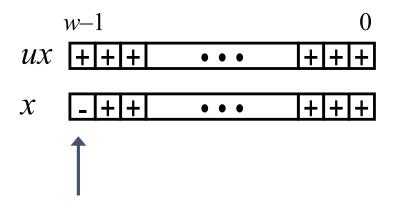


	Ingianad	
_	Insigned	
	0	
	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
	11	
	12	
	13	
	14	
	15	

Relation between Signed & Unsigned



Maintain Same Bit Pattern



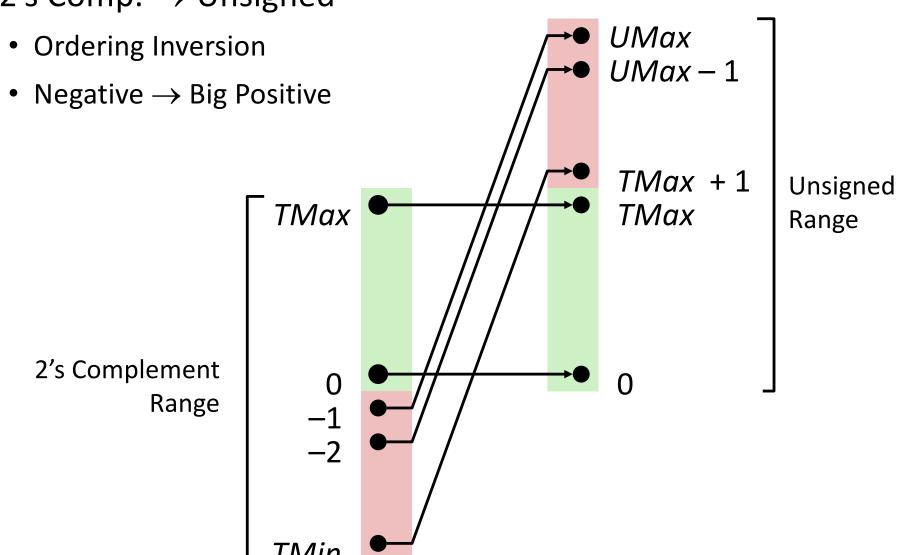
Large negative weight

becomes

Large positive weight

Conversion Visualized

• 2's Comp. → Unsigned



Signed vs. Unsigned in C

- Constants
 - By default, considered as signed integers
 - Unsigned if have "U" as suffix
 0U, 4294967259U
- Casting
 - Explicit casting between signed & unsigned same as U2T and T2U

```
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```

Implicit casting also occurs via assignments and procedure calls

```
tx = ux;

uy = ty;
```

Casting Surprises

- Expression Evaluation
 - If there is a mix of unsigned and signed in single expression, signed values implicitly cast into unsigned values
 - Including comparison operations <, >, ==, <=, >=
 - Examples for W = 32: TMIN = -2,147,483,648, TMAX = 2,147,483,647

Constant ₁	Constant ₂	Relation	Evaluation
0	0υ	==	unsigned
-1	0	<	signed
-1	OΠ	unsigned	
2147483647	-2147483648	>	signed
2147483647U	-2147483648	<	unsigned
-1	-2	>	signed
(unsigned) -1	-2	>	unsigned
2147483647	2147483648U	<	unsigned
2147483647	(int) 2147483648U	>	signed

Summary

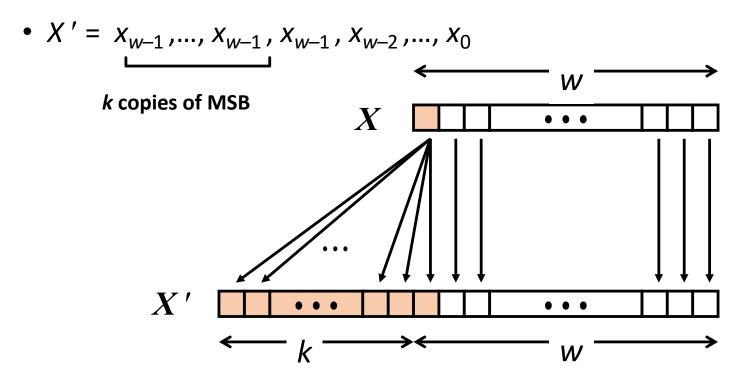
- - Bit pattern is maintained
 - But reinterpreted
 - Can have unexpected effects: adding or subtracting 2^w
 - Expression containing signed and unsigned int
 - (signed) int is cast into unsigned int!!

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Sign Extension

- Task:
 - Given w-bit signed integer x
 - Convert it to w+k-bit integer with same value
- Rule:
 - Make k copies of sign bit:



Sign Extension Example

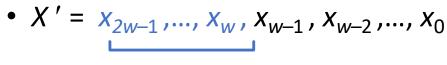
```
short int x = 15213;
int     ix = (int) x;
short int y = -15213;
int     iy = (int) y;
```

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
ix	15213	00 00 3B 6D	00000000 00000000 00111011 01101101
У	-15213	C4 93	11000100 10010011
iy	-15213	FF FF C4 93	11111111 11111111 11000100 10010011

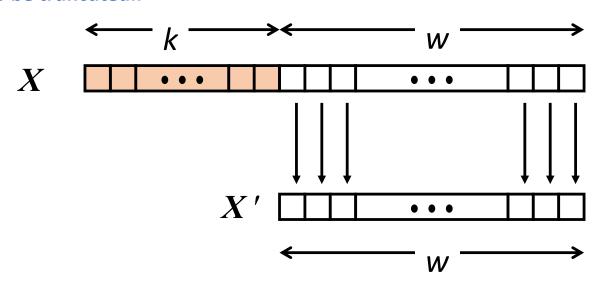
- Converting from smaller to larger integer data type
- C automatically performs sign extension

Truncate

- Task:
 - Given **w+k**-bit signed integer x
 - Convert it to w-bit integer → Values may change!!
- Rule:
 - k-bit will be removed (as overflowed k-bit)



To be truncated..



Summary

• Expanding, Truncating: Basic Rules

- Expanding (e.g., short int to int)
 - Unsigned: zeros added
 - Signed: sign extension
 - Both yield expected result
- Truncating (e.g., unsigned to unsigned short)
 - Unsigned/signed: bits are truncated
 - Result reinterpreted
 - Unsigned: mod operation
 - Signed: similar to mod
 - For small numbers yields expected behavior

Today: Bits, Bytes, and Integers

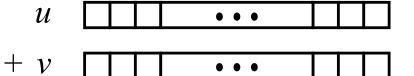
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Unsigned Addition

Operands: w bits

True Sum: w+1 bits

Discard Carry: w bits



$$u + v$$

 $UAdd_{w}(u, v)$



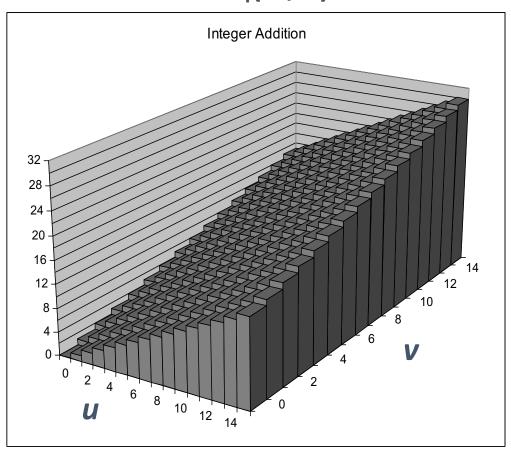
- Standard Addition Function
 - Ignores carry output
- Implements Modular Arithmetic

$$s = UAdd_w(u, v) = u + v \mod 2^w$$

Visualizing (Mathematical) Integer Addition

- Integer Addition
 - 4-bit integers *u*, *v*
 - Compute true sum $Add_4(u, v)$
 - Values increase
 linearly with u and v
 - Forms planar surface

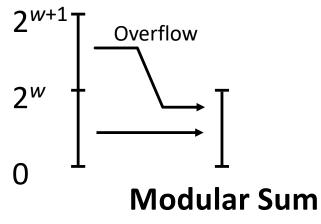
$Add_4(u, v)$

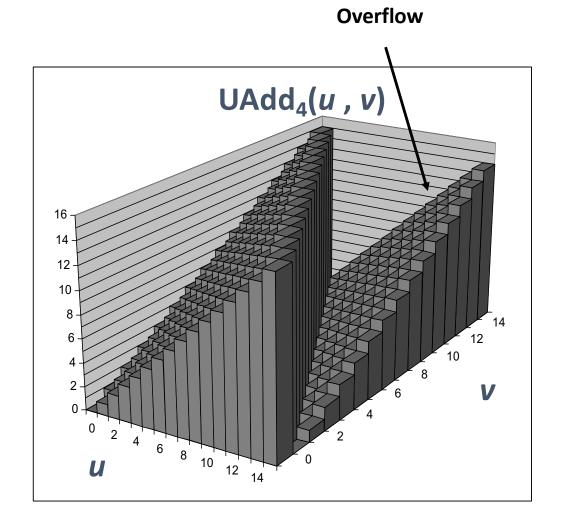


Visualizing Unsigned Addition

- Wraps Around
 - If true sum $\geq 2^w$
 - At most once

True Sum





Two's Complement Addition

- TAdd and UAdd have Identical bit-level behavior (why?)
 - Signed vs. unsigned addition in C:

```
int s, t, u, v;
s = (int) ((unsigned) u + (unsigned) v);
t = u + v
```

• Will give s == t

TAdd Overflow

Functionality

- True sum requires w+1 bits
- Drop off MSB
- Treat remaining bits as 2's comp. integer

Example)

True Sum

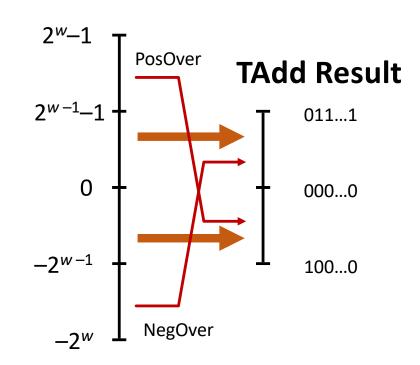
0 111...1

0 100...0

0 000...0

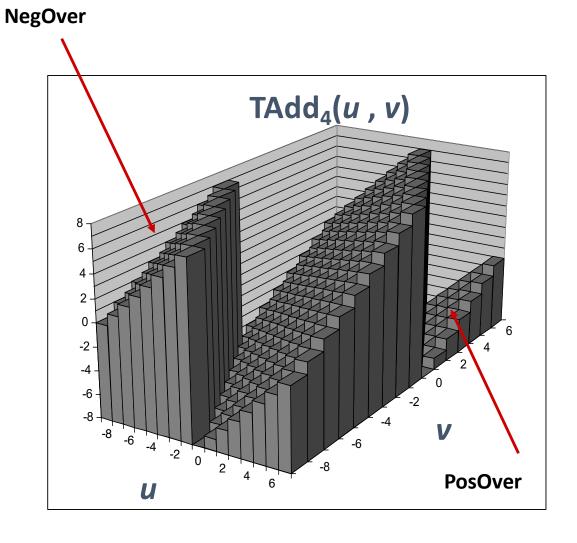
1 011...1

1 000...0



Visualizing 2's Complement Addition

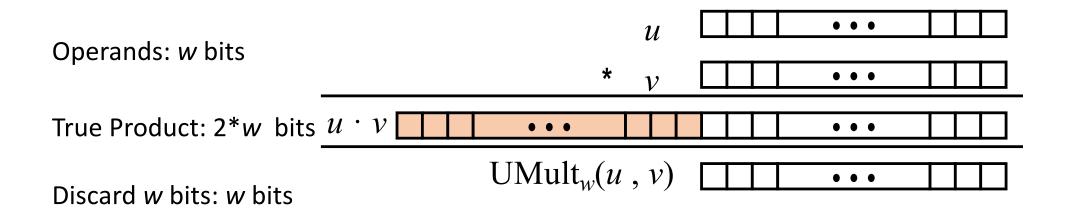
- Values
 - 4-bit two's comp.
 - Range from -8 to +7
- Wraps Around
 - If sum $\geq 2^{w-1}$
 - Becomes negative
 - At most once
 - If sum $< -2^{w-1}$
 - Becomes positive
 - At most once



Multiplication

- Goal: Computing Product of w-bit numbers x, y
 - Either signed or unsigned
- But, exact results can be bigger than w bits
 - Unsigned: up to 2w bits
 - Result range: $0 \le x * y \le (2^w 1)^2 = 2^{2w} 2^{w+1} + 1$
 - Two's complement min (negative): Up to 2w-1 bits
 - Result range: $x * y \ge (-2^{w-1})*(2^{w-1}-1) = -2^{2w-2} + 2^{w-1}$
 - Two's complement max (positive): Up to 2w bits, but only for $(TMin_w)^2$
 - Result range: $x * y \le (-2^{w-1})^2 = 2^{2w-2}$
- So, maintaining exact results...
 - would need to keep expanding word size with each product computed
 - is done in software, if needed
 - e.g., by "arbitrary precision" arithmetic packages

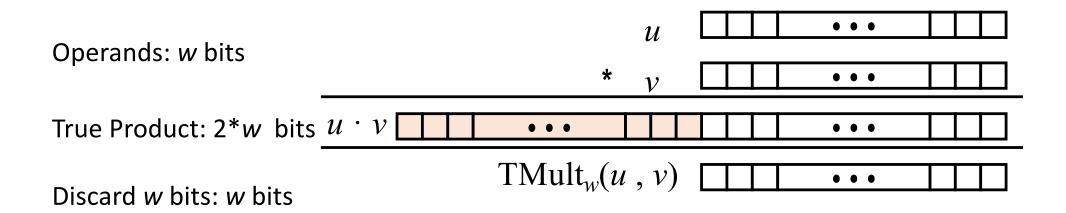
Unsigned Multiplication in C



- Standard Multiplication Function
 - Ignores high order w bits
- Implements Modular Arithmetic

$$UMult_w(u, v) = u \cdot v \mod 2^w$$

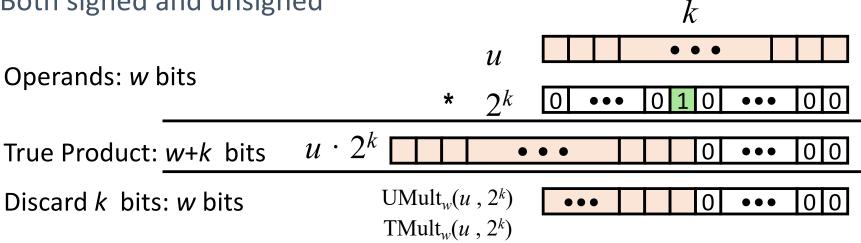
Signed Multiplication in C



- Standard Multiplication Function
 - Ignores high order w bits
 - Some of which are different for signed vs. unsigned multiplication
 - Lower bits are the same

Power-of-2 Multiply with Shift

- Operation
 - $\mathbf{u} \ll \mathbf{k}$ gives $\mathbf{u} * \mathbf{2}^k$
 - Both signed and unsigned



Examples

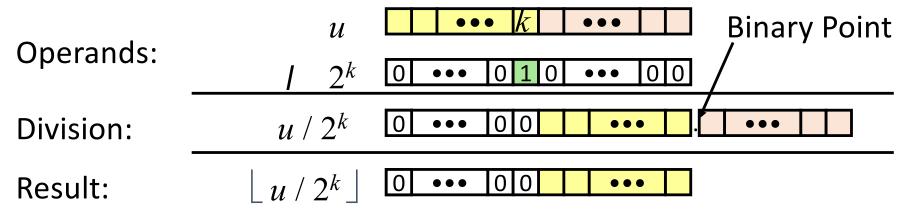
$$u << 3 == u * 8$$

 $(u << 5) - (u << 3) == u * 24$

- Most machines shift and add faster than multiply
 - Compiler generates this code automatically

Unsigned Power-of-2 Divide with Shift

- Quotient of Unsigned by Power of 2
 - $\mathbf{u} \gg \mathbf{k}$ gives $\lfloor \mathbf{u} / 2^k \rfloor$
 - Uses *logical right shift*



	Division	Computed	Hex	Binary		
x	15213	15213	3B 6D	00111011 01101101		
x >> 1	7606.5	7606	1D B6	00011101 10110110		
x >> 4	950.8125	950	03 B6	00000011 10110110		
x >> 8	59.4257813	59	00 3B	00000000 00111011		

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Arithmetic: Basic Rules

Addition:

- Unsigned/signed: Normal addition followed by truncate, same operation on bit level
- Unsigned: addition mod 2^w
 - Mathematical addition + possible subtraction of 2^w
- Signed: modified addition mod 2^w (result in proper range)
 - Mathematical addition + possible addition or subtraction of 2^w

• Multiplication:

- Unsigned/signed: Normal multiplication followed by truncate, same operation on bit level
- Unsigned: multiplication mod 2^w
- Signed: modified multiplication mod 2^w (result in proper range)

When to Use Unsigned

- Don't use without understanding implications
 - Easy to make mistakes

```
unsigned i;
for (i = cnt-2; i >= 0; i--)
a[i] += a[i+1];
```

Can be very subtle

```
#define DELTA sizeof(int)
int i;
for (i = CNT; i-DELTA >= 0; i-= DELTA)
. . . .
```

Counting Down with Unsigned

Proper way to use unsigned as loop index

```
unsigned i;
for (i = cnt-2; i < cnt; i--)
  a[i] += a[i+1];</pre>
```

- See Robert Seacord, Secure Coding in C and C++
 - C Standard guarantees that unsigned addition will behave like modular arithmetic
 - $0-1 \rightarrow UMax$
- Even better

```
size_t i;
for (i = cnt-2; i < cnt; i--)
a[i] += a[i+1];</pre>
```

- Data type **size t** defined as unsigned value with length = word size
- Code will work even if cnt = UMax
- What if cnt is signed and < 0?

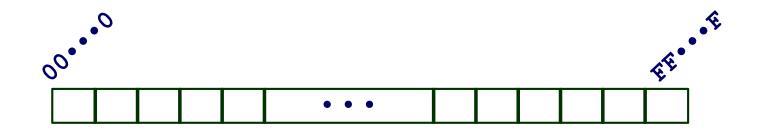
When to Use Unsigned (Con't)

- Do use when performing *modular* arithmetic
 - Multi-precision arithmetic
- Do use when using bits to represent sets
 - Logical right shift, no sign extension

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Byte-Oriented Memory Organization



- Programs refer to data by address
 - Conceptually, envision it as a very large array of bytes
 - In reality, it's not, but can think of it that way
 - An address is like an index into that array
 - A pointer variable stores an address
- Note: system provides private address spaces to each process

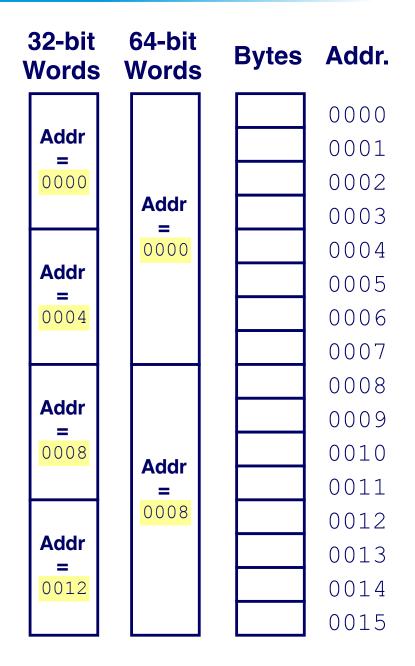
Machine Words

- Any given computer has a fixed-size word
 - Nominal size of integer-valued data and addresses
- Decades ago, most machines used 32 bits (4 bytes) as word size
 - Limits addresses to 4GB (2³² bytes)
- Most machines now have 64-bit word size
 - Potentially, could have 18 EB (exabytes) of addressable memory
- Machines still support multiple data formats
 - Fractions or multiples of word size
 - Always integral number of bytes



Word-Oriented Memory Organization

- Addresses Specify Byte Locations
 - Address of first byte in word
 - Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



Byte Ordering

 So, how are the bytes within a multi-byte word ordered in memory?

- Conventions
 - Big Endian: Sun, PPC Mac, Internet
 - Least significant byte has highest address
 - Little Endian: x86, ARM processors running Android, iOS, and Windows
 - Least significant byte has lowest address

Byte Ordering Example

- Example
 - Variable x has 4-byte value of 0x01234567
 - Address given by &x is 0x100

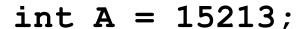
Big Endian			0x100	0x101	0x102	0 x 103	
			01	23	45	67	
Little Endian		0 x 100	0x101	0x102	0x103		
			67	45	23	01	

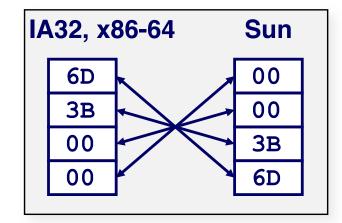
Representing Integers

Decimal: 15213

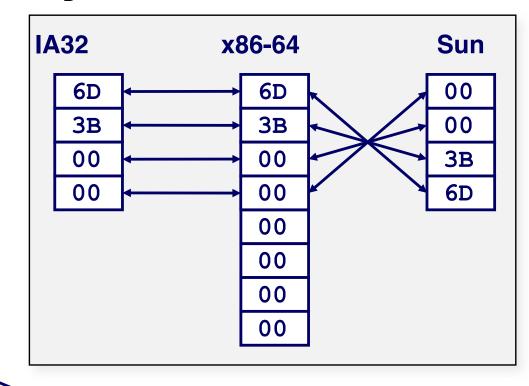
Binary: 0011 1011 0110 1101

Hex: 3 B 6 D

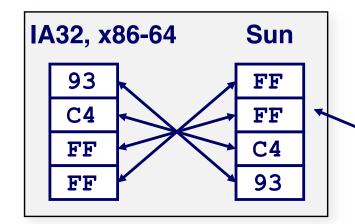




long int C = 15213;



int B = -15213;



Two's complement representation

Examining Data Representations

- Code to print byte-representation of data
 - Casting pointer to unsigned char* allows treatment as a byte array

```
typedef unsigned char *pointer;

void show_bytes(pointer start, size_t len) {
    size_t i;
    for (i = 0; i < len; i++)
        printf("%p\t%.2x\n",start+i, start[i]);
    printf("\n");
}</pre>
```

printf directives:

%**p**: Print pointer

%x: Print Hexadecimal

show bytes Execution Example

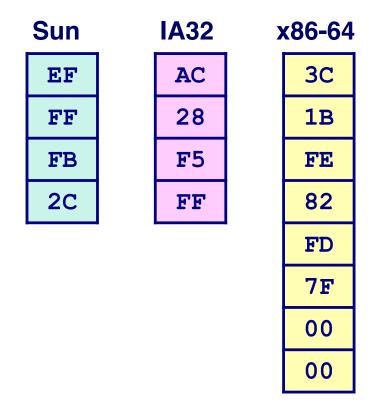
```
int a = 15213;
printf("int a = 15213;\n");
show_bytes((pointer) &a, sizeof(int));
```

Result (Linux x86-64):

```
int a = 15213;
0x7fffb7f71dbc 6d
0x7fffb7f71dbd 3b
0x7fffb7f71dbe 00
0x7fffb7f71dbf 00
```

Representing Pointers

```
int B = -15213;
int *P = &B;
```



- Different compilers & machines assign different locations to objects
- Even get different results each time run program

Representing Strings

- Strings in C
 - Represented by array of characters
 - Each character encoded in ASCII format
 - Standard 7-bit encoding of character set
 - Character "0" has code 0x30
 - Digit *i* has code 0x30+*i*
 - String should be null-terminated
 - Final character = 0
- Compatibility
 - Byte ordering not an issue?

char S[6] = "18213";

	ASCII control			ASCII printable						
characters				characters						
00 NULL (Null character)				32	space	64	@	96		
01	SOH	(Start of Header)		33	!	65	Ā	97	а	
02	STX	(Start of Text)		34		66	В	98	b	
03	ETX	(End of Text)		35	#	67	С	99	С	
04	EOT	(End of Trans.)		36	\$	68	D	100	d	
05	ENQ	(Enquiry)		37	%	69	E	101	е	
06	ACK	(Acknowledgement)		38	&	70	F	102	f	
07	BEL	(Bell)		39		71	G	103	g	
08	BS	(Backspace)		40	(72	Н	104	h	
09	HT	(Horizontal Tab)		41)	73	- 1	105	i.	
10	LF	(Line feed)		42	*	74	J	106	j	
11	VT	(Vertical Tab)		43	+	75	K	107	k	
12	FF	(Form feed)		44	,	76	L	108	- 1	
13	CR	(Carriage return)		45	-	77	M	109	m	
14	SO	(Shift Out)		46		78	N	110	n	
15	SI	(Shift In)		47	1	79	0	111	0	
16	DLE	(Data link escape)		48	0	80	Р	112	р	
17	DC1	(Device control 1)		49	1	81	Q	113	q	
18	DC2	(Device control 2)		50	2	82	R	114	r	
19	DC3	(Device control 3)		51	3	83	S	115	S	
20	DC4	(Device control 4)		52	4	84	Т	116	t	
21	NAK	(Negative acknowl.)		53	5	85	U	117	u	
22	SYN	(Synchronous idle)		54	6	86	V	118	٧	
23	ETB	(End of trans. block)		55	7	87	W	119	w	
24	CAN	(Cancel)		56	8	88	Х	120	х	
25	EM	(End of medium)		57	9	89	Y	121	У	
26	SUB	(Substitute)		58	:	90	Z	122	z	
27	ESC	(Escape)		59	;	91	[123	{	
28	FS	(File separator)		60	<	92	1	124	Ļ	
29	GS	(Group separator)		61	=	93]	125	}	
30	RS	(Record separator)		62	>	94	^	126	~	
31	US	(Unit separator)		63	?	95	_			
127	DEL	(Delete)								

Integer C Puzzles

Initialization

```
\rightarrow ((x*2) < 0)
• x < 0
• ux >= 0
• x \& 7 == 7 \rightarrow (x<<30) < 0
• ux > -1
                  \rightarrow -x < -y
\cdot x > y
• x * x >= 0
• x > 0 \&\& y > 0 \implies x + y > 0
                   \rightarrow -x <= 0
\cdot x >= 0
                \rightarrow -x >= 0
• x <= 0
• (x|-x)>>31 == -1
• ux >> 3 == ux/8
• x >> 3 == x/8
 x \& (x-1) != 0
```