Review 4−1

- Hajin Ju, 2024062806

Problem 1

Show that the solution of $T(n) = 2T(\lfloor n/2 \rfloor) + n$ is $O(n \lg n)$ by the substitution method. (Show the inductive step only.)

Solution 1

- inductive step

$$T(n) \le cn \lg n, \quad \text{(for some } c > 0,\ n > n_0)$$

We can assume the hypothesis($T(n) = O(n \lg n)$) holds for all positive int smaller than n.
then,

$$\begin{aligned} T(\lfloor n/2 \rfloor) \le c\lfloor n/2 \rfloor \lg \lfloor n/2 \rfloor &\le c(n/2) \lg(n/2) \\ &= c(n/2)(\lg n - \lg 2) \end{aligned}$$

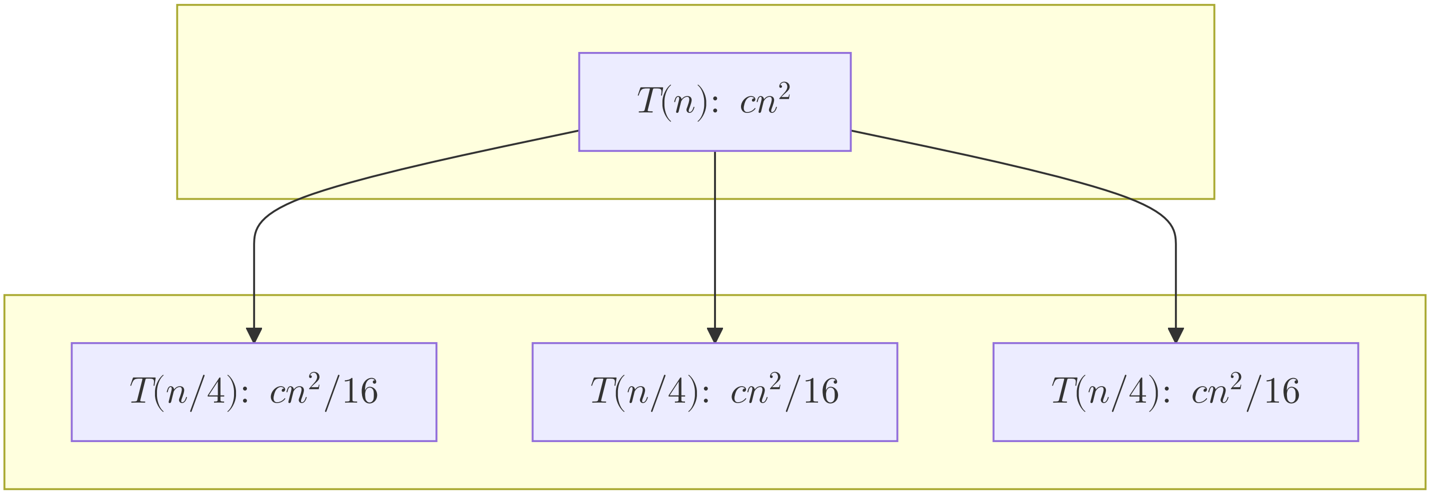
$$\begin{aligned} T(n) &= 2T(\lfloor n/2 \rfloor) + n \le cn(\lg n - \lg 2) + n \\ &= cn \lg n - cn \lg 2 + n \\ &= cn \lg -cn + n \\ &\le cn \lg n \end{aligned}$$

therefore, $T(n) = O(n \lg n)$

Problem 2

Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = 3T(\lfloor n/4 \rfloor) + \theta(n^2)$.

Solution 2



- level 0
  - cn^2
- level 1
  - $\frac{3}{16}cn^2$
- level k
  - $(\frac{3}{16})^k cn^2$
- level $\log_4 n$
  - $n^{\log_4 3}$

therefore, total cost is

$$\begin{aligned} T(n) &= \sum_{i=0}^{\log_4 n-1} (\frac{3}{16})^i cn^2 + \Theta(n^{\log_4 3}) \\ &< \sum_{i=0}^{\infty} (\frac{3}{16})^i cn^2 + \Theta(n^{\log_4 3}) \\ &= \frac{16}{13}cn^2 + \Theta(n^{\log_4 3}) = O(n^2) \end{aligned}$$