Review 8–5

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Problem 1

The minimum number of scalar multiplications for computing $A_i A_{i+1} \ldots A_j$, denoted by $m[i, j]$, is as follows. Fill in the blanks.

Solution 1

$$m[i, j] = \begin{cases} 0 & \text{if } i = j \\ \min_{i \le k < j} \left(m[i][k] + m[k+1][j] + p_{i-1} p_k p_j \right) & \text{if } i < j \end{cases}$$

Problem 2

Compute (a)$m[2, 5]$ and (b)$s[2, 5]$ in the following example and parenthesize (c) the prodct $A_1 A_2 A_3 A_4 A_5 A_6$ fully to minimize the number of scalar multiplications.

m	1	2	3	4	5	6
1	0	15750	7875	9375	11875	15125
2		0	2625	4375	(a)	10500
3			0	750	2500	5375
4				0	1000	3500
5					0	5000
6						0

s	2	3	4	5	6
1	1	1	3	3	3
2		2	3	(b)	3

s	2	3	4	5	6
3			3	3	3
4				4	5
5					5
6					

name	matrix dimension
A_1	30×35
A_2	35×15
A_3	15×5
A_4	5×10
A_5	10×20
A_6	20×25

Solution 2

index	p
0	30
1	35
2	15
3	5
4	10
5	20
6	25

(a).

$$m[2,5] = \left(\min \begin{cases} m[2][2] + m[3][5] + p[1][2][3] & = 13000 \\ m[2][3] + m[4][5] + p[1][3][5] & = 7125 \\ m[2][4] + m[5][5] + p[1][4][5] & = 11375 \end{cases} \right) = 7125$$

(b). therefore $s[2,5] = 3$

(c).

$$(A_1(A_2A_3))((A_4A_5)A_6)$$

Problem 3

Fill in the blanks in the following pseudocode for `MATRIX-CHAIN-ORDER`.

Solution 3

```
MATRIX-CHAIN-ORDER (p)
    let m[1..n, 1..n] and s[1..(n-1), 2..n] be new tables
    for i = 1 to n
        m[i, i] = 0
    for l = 2 to n
        for i = 1 to n - l + 1
            j = i + l - 1
            m[i, j] = inf
            for k = i to j - 1
                q = m[i][k] + m[k + 1][j] + p[i-1] * p[k] * p[j]
                if q < m[i, j]
                    m[i, j] = q
                    s[i, j] = k
    return m and s
```

Problem 4

Fill in the blanks in the following pseudocode for `PRINT-OPTIMAL-PARENS`.

Solution 4

```
PRINT-OPTIMAL-PARENS (s, i, j)
    if i == j
        print "A_i"
    else print "("
        PRINT-OPTIMAL-PARENS(s, i, s[i, j])
        PRINT-OPTIMAL-PARENS(s, s[i, j] + 1, j)
        print ")"
```