

# Review 3

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## Problem 1

Write ☐ if an entry is true or ☐ otherwise.

### Solution 1

	$O(n \lg n)$	$\Omega(n \lg n)$	$\Theta(n \lg n)$
$\lg n$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$n$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$n \lg n$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$n \lg^2 n$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$n^2$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

## Problem 2

Show  $3n + 1 = O(n^2)$  by the definition of  $O$ .

### Solution 2

A function  $f(n) = O(g(n))$  if there exist constants  $c \geq 0$  and  $n_0 \geq 0$ , s.t.

$$n \geq n_0 \Rightarrow \leq |f(n)| \leq c|g(n)|$$

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let  $g(n) = n^2$

let  $f(n) = 3n + 1$

suppose  $c = 4, n_0 = 1$

and then, for all  $n \geq 1 \rightarrow |3n + 1| \leq 4n^2$

therefore,  $3n + 1 = O(n^2)$  by the above definition.

## Problem 3

Write asymptotic notations that satisfy each relation and explain why.

1. Transitivity
2. Reflexivity
3. Symmetry

## Solution 3

1. Transitivity

- $O$  is transitive  
because  $f(n) = O(g(n))$  and  $g(n) = O(h(n))$  implies  $f(n) = O(h(n))$   
there must exists  $n_0 \geq 0$ , s.t.  $n \geq n_0 \Rightarrow f(n) \leq c_0 g(n) \leq c_1 c_0 h(n)$
- $\Omega$  is transitive  
because  $f(n) = \Omega(g(n))$  and  $g(n) = \Omega(h(n))$  implies  $f(n) = \Omega(h(n))$   
there must exists  $n_0 \geq 0$ , s.t.  $n \geq n_0 \Rightarrow f(n) \geq c_0 g(n) \geq c_1 c_0 h(n)$
- $\Theta$  is transitive  
because  $f(n) = \Theta(g(n))$  and  $g(n) = \Theta(h(n))$  implies  $f(n) = \Theta(h(n))$

$$f(n) = O(h(n)) \wedge f(n) = \Omega(h(n))$$

2. Reflexivity

- $O$  is reflexive  
because  $f(n) = O(f(n))$  where  $c = 1$
- $\Omega$  is reflexive  
because  $f(n) = \Omega(f(n))$  where  $c = 1$
- $\Theta$  is reflexive

- because  $f(n) = \Theta(f(n))$

### 3. Symmetry

- $O$  is **not** symmetric  
because  $f(n) = O(g(n))$  does not imply  $g(n) = O(f(n))$   
for example,  $n = O(n^2)$  cannot imply  $n^2 = O(n)$
- $\Omega$  is **not** symmetric  
because  $f(n) = \Omega(g(n))$  does not imply  $g(n) = \Omega(f(n))$   
for example,  $n^2 = \Omega(n)$  cannot imply  $n = \Omega(n^2)$
- $\Theta$  is symmetric  
because  $f(n) = \Theta(g(n))$  implies  $g(n) = \Theta(f(n))$